

# Psychological Bulletin

## PERCEPTUAL INDEPENDENCE:

### DEFINITIONS, MODELS, AND EXPERIMENTAL PARADIGMS<sup>1</sup>

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A logical analysis of the problem of the experimental study of perceptual independence is undertaken. Necessary distinctions are: between the definition of independence as zero correlation and its definition as performance parity, between normative models of perceptual process and models of perceptual state, between experimental designs which use orthogonal stimulus inputs and those which use correlated inputs. In addition, it is often necessary to specify an organismic decision rule, which can be of many types and can occur with respect to process or to state. Implications of this analysis are: Perceptual independence is not a unitary concept; it can be studied most effectively with experiments using orthogonal inputs; when correlated stimulus inputs are used, so many underlying assumptions have to be made that it is almost impossible not to find a model which satisfies both the concept of independence and any given experimental result; state independence is intimately related to process independence; and search for the locus of interaction is probably a more useful approach than trying to determine whether independence exists.

A major problem under attack by experimental psychologists in recent years is the determination of how the organism deals with and makes use of multiple sources of information. To illustrate some of the many types of experimental question being asked: If two tones are presented in succession to one ear, is the resultant probability of detection greater than if just one tone is presented? If a tone is presented simultaneously to the

two ears, is the threshold lower than if the tone is presented to one ear alone? If a set of stimuli covaries in two attributes, such as color and size, are absolute judgments of these stimuli more accurate than if they vary in just one attribute; that is, does redundant stimulus information improve perceptual performance? Does the probability of correct identification of a visual form improve if it is presented simultaneously to two parts of the same eye, or to the two eyes together, or perhaps to the same eye in successive time periods? In searching for a target form among many different forms all on a single display at the same time, can the organism simultaneously search for this target in all locations at once, or must it successively scan the different target locations? Still further, if we present different speech passages to the two ears, can the organism show adequate perceptual response to each without interference from the other? Or, as a last example, suppose the prior verbal context for a word is given as one source of information, and the

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word is then presented under adverse stimulus conditions, how does performance with both sources of information relate to performance with either alone?

In all of these examples there is a similar conceptual problem: Does the organism process the two or more sources or channels of information independently? This question concerns primarily the perceptual process, but the question of independence must necessarily involve the nature of stimulus independence, response independence, and decision independence, since all of these conceptually separable processes are involved in any single experiment on the problem.

Even though all of the experimental questions mentioned previously are actual cases from the experimental literature (most referenced in this paper), the purpose of the present paper is not to review or criticize this experimental literature but to provide a logical analysis of the problem itself. For this purpose entirely hypothetical data are used in the illustrations, so that special aspects of the problem can be illustrated more clearly than is possible with any real experiment, which nearly always will involve more than one aspect of the problem of independence. The use of such idealized data does not mean that the problems do not exist in real experiments but only that the several aspects of the problem can be easily separated with idealized data.

#### THE ROLE OF NORMATIVE MODELS

A common and logically sound method of dealing with this problem is to use a normative model (ordinarily mathematical) whose purpose is to establish what ideal performance should be with the assumption of independence, to provide a base line against which to compare actual performance. Without such a base line, any given experimental result is so crudely qualitative that we obtain little generality to our understanding of the problem. Thus the normative model provides meaningful interpretation of any given experimental finding.

The need for models in understanding perceptual independence has clearly been recognized, since it is a rare experiment in

this area whose results have not been evaluated against some model of independence. The rather special role of the normative model has perhaps been less well understood, however, since models often are accepted or rejected on the basis of the fit of the data to a particular model. But the special role of the normative model is not to describe a particular perceptual process but to provide a norm against which results are evaluated. Thus we should not ask whether the model is right or wrong but rather to what extent the experimental results conform to the criterion of independence which the particular model implies.

This is not to say that normative models cannot be wrong; rather, it is to say that the basis of their being wrong (or right) is not so much in fitting data as in correctly reflecting the underlying concepts or processes which the experimenter intends them to reflect. Further, normative models may be more or less useful not insofar as data fit a particular model but insofar as the model helps clarify the nature of the process when the data do not fit the model. Thus, not only must the normative model indicate expected values, it must be expressed in such a form that we can understand why the data do not fit the model. Simple rejection of a null hypothesis is much less valuable than understanding the nature of the process which leads to rejection of the hypothesis.

In looking over the many models used in the study of perceptual independence and the experimental situations to which they are applied, it seems that several sources of confusion have led both to a proliferation of models and to a lack of emergence of a few basic explanatory concepts. These particular confusions do not concern whether various perceptual processes are in fact independent. Instead, the confusions concern logical and rational aspects of the very question which is being asked, and these are the things discussed here. There are three basic factors which need to be considered: (a) the definition of independence that is used, (b) the type of model that is used, and (c) the experimental paradigm within which the problem is investigated. The present paper considers

these factors and the interrelations between them in the interest of providing some clarification of the study of perceptual independence.

#### INDEPENDENCE: CORRELATION OR PARITY

There are two basic definitions of independence which are used, that of zero correlation and that of performance parity. While these two definitions are related, both concepts can be operative in a particular experimental situation, and the two different definitions of independence are themselves capable of being independent.

#### *Zero Correlation*

The most basic definition of independence in any formal sense is that two variables or sets of events are uncorrelated. We say that outcomes of coin tosses are independent if the outcome on Trial  $n$  is uncorrelated with the outcome on Trial  $n - 1$ . We say that two tests are independent if their scores do not correlate across subjects. Even the commonly used cross-multiplication of probabilities to establish independence is essentially a zero-correlation criterion, since the cross product is the value of joint occurrence which obtains with zero correlation.

Insofar as possible, metric definitions of zero correlation are desirable, since with such measurable correlations exact limits of maximum and minimum correlation can be specified. Nevertheless, there are many experimental problems to which correlation as an exact measure is not applicable, although the idea of independence and even of correlation at a conceptual level is. For example, we speak of two speech passages as uncorrelated if they are simply completely different passages. If the same passage in the same voice is presented to, say, the two ears simultaneously, we can say that the two ears are receiving correlated messages; but if the passages are simply unrelated, it seems more reasonable to talk about lack of correlation rather than zero correlation, since clear metric specification of correlation is not possible in this case.

#### *Performance Parity*

An alternative definition of independence is that of performance parity. Briefly stated, the performance-parity criterion is that two perceptual processes are considered independent if performance on two perceptual tasks carried out simultaneously is the same as the sum of the performances on each task carried out separately. The two tasks might require dealing with information in two modalities, or two subaspects of the same modality, or two dimensions of a set of stimuli. With certain tasks, this criterion of independence seems intuitively reasonable, although in others it does not. Certainly it is worth considering why performance parity should be used as a criterion of independence at all, and how it has come to be accepted as such a criterion if the basic definition of independence is zero correlation.

#### *Relations between Correlation and Parity*

There are definite relations between the performance-parity and the zero-correlation criteria of independence that have allowed substitution of the derived parity criterion for the direct correlation criterion. Consider, for example, the relation between the variance of a sum (or difference) and the variance of the components making up the sum. If we have just two variables,  $X$  and  $Y$ , then

$$S^2_{(X+Y)} = S^2_X + S^2_Y + 2r_{XY}S_XS_Y, \quad [1]$$

where  $S^2$  is a variance. The same relation holds for the difference of two variables except that the correlation term is negative. This relation says, in words, that the variance of a sum equals the sum of the variances of the individual variables if and only if the correlation between the two variables is zero. Thus the criterion of independence as zero correlation becomes a criterion of parity of the variances; and in many situations, it is easier to test the parity consequence than to measure zero correlation. For example, if we have two sets of scores with known variances and also know the variance of their sums, we can say that the two sets of scores are uncorrelated if parity of the variance holds. Thus we can say that the two sets of scores are

independent without actually calculating the correlation between the sets.

Completely similar relations exist with the information metric. Using Garner's (1962) notation,

$$U(X,Y) = U(X) + U(Y) - U(X:Y), \quad [2]$$

which in words states that the joint uncertainty (or information) of variables  $X$  and  $Y$  equals the sum of the uncertainties of variables  $X$  and  $Y$  separately minus the contingent uncertainty between  $X$  and  $Y$ . Thus parity of information holds if and only if there is no contingent uncertainty (a kind of nonmetric correlation) between  $X$  and  $Y$ .

Further examples of the intimate relation between correlation and parity come from the multiple-predictor situation. For example, in a bivariate analysis of variance (i.e., two predictor variables plus one criterion variable), the total variance of the criterion equals the sum of the variances due to the two predictor variables only if there is no interaction, which is a kind of correlation between the two predictor variables in terms of their effects on the criterion. And a squared multiple-correlation coefficient equals the sum of the squared correlations of the criterion variable with each predictor variable if and only if the two predictor variables are themselves uncorrelated, that is, independent. Once again, completely analogous relations exist with the information metric (see Garner & McGill, 1956).

Thus there is a clear formal rationale which lies behind the use of performance parity as an indirect criterion of independence. What has happened, however, is that the performance-parity criterion has become quite independent of the formal rationale; it has become functionally autonomous, so to speak. This functional autonomy is entirely reasonable, since performance parity can be measured in situations where there is no real possibility of measuring a correlation of any kind, and yet where the need to ask about perceptual independence is more than apparent. To illustrate, suppose we present different speech in two ears and ask for total comprehension. If both passages can be fully understood (as determined by some per-

formance measure), we say that parity has been satisfied and infer that the processes are independent. Yet there is no meaningful way to measure a correlation coefficient of any kind. Or, alternatively, we present a verbal context plus tachistoscopic exposure of a word, and we assume independence of the two sources of information if somehow performance with both sources can be shown to be the sum of the performances when each source of information is separate.

Thus there is a real need for a performance-parity criterion of independence, if it can be applied without serious distortion of the fundamental concept of independence. However, the parity criterion should be used only when there is no reasonable possibility of measuring correlation directly. In some circumstances, it is entirely reasonable to ask about perceptual independence both as zero correlation and as performance parity in the same experimental situation.

#### *Parity and Process Separability*

Both the correlation criterion and the parity criterion of independence can usually, although not always, be applied to data in some form; that is, they are operationally definable with respect to directly obtained experimental data and derived properties of these data. Yet even though these concepts can be considered external to the organism under study, the aim of psychologists is ultimately to understand the internal processes responsible for these external concepts.

Related to this entire question of independence is the organismic concept of process separability, which is simply the assumption that two processes act separately in the organism and thus can go on independently of each other. Such an organismic concept, with its connotation of process interaction, can be related conceptually either to the correlation or the parity criterion of independence; that is, to say that two processes interact could mean that they are in some way correlated or that each process in some way limits performance with respect to the other, perhaps by sharing a common channel. In actual practice, however, the concept of process separability is nearly always related to the parity criterion

of independence and is most frequently applied in situations where correlation cannot be directly measured.

There are many terms used in reference to the general idea of process separability. In recent years, the distinction between parallel and serial processing has received considerable attention (see Neisser, 1967; Posner & Mitchell, 1967, among others). The notion of parallel processing is that two perceptual acts can occur simultaneously with performance parity. Alternatively, the serial processing idea is that only one process can occur at a time, so that some form of process time sharing is necessary. Exact interpretation of these two concepts in terms of process independence is actually rather tricky, since models which assume parallel processing do not invariably lead to performance parity (see Egeth, 1966). In addition, the assumption of a short-term memory which preserves information completely makes possible a serial model with no loss of parity.

Concepts such as serial and parallel processing, or the more general idea of process separability, even though they may be useful in their own right, often produce only confusion when related to the problem of perceptual independence. Certainly at times there is real contradiction between the separability idea and performance parity. Lockhead (1966), for example, introduced the idea of stimulus integrality (quite the opposite of separability) to explain the circumstances under which performance parity will occur in an absolute-judgment task involving multiple stimulus dimensions. Other authors, for example, Shepard (1964), Hyman and Well (1968), and Handel (1967) introduced the separability idea to explain greater summative effects of two stimulus dimensions in judgments of stimulus differences. Thus we have the conflicting claims that separability is required for greater differentiation and that it prevents performance parity. And yet each seems intuitively reasonable in its own context.

As another illustration, consider Broadbent's (1958) concept of a filter process, a process in which one of two input channels is completely rejected while the other is passed through. Such a concept suggests process

separability, since clearly the organism can prevent interaction between two channels. At the same time, if the organism does reject one while accepting the other, we have evidence of serial processing, a concept which suggests nonseparability of process. To some extent, these apparent confusions can be clarified by distinguishing between process separability and channel separability.

However, at the present time it is not clear that organismic concepts related to either process or channel separability (and thus appearing to be intimately related to the question of perceptual independence) are in a position to clarify the problem of perceptual independence. And, conversely, it is not clear that concepts of perceptual independence are in a position to clarify the problem of channel or process separability.

#### TYPE OF MODEL

With regard to type of model, at least three kinds of distinction need to be made. Furthermore, it is not always clear that the distinction should be attributed to the type of model rather than to the nature of the process assumed, since if the model has any hope of reflecting psychological reality (i.e., processes inside the organism), at least some of the properties of the model will be reflecting properties of the organism. Where we choose to make the distinction is, however, less important than that we make it and recognize the consequences of the distinction as related to our understanding of perceptual independence.

#### *Process versus State Models*

The first and most important distinction is between models concerned with independence of the direct process itself and those models which are concerned with independence of the states of the possibly separate channels. To illustrate, we can ask whether apparent brightness is influenced by color, and this is a question which concerns correlations between judgments of brightness and variations in color. If there is no correlation, we say that brightness is independent of color. Thus this question concerns whether the color and brightness processes are correlated in the

sense that the relevant processes themselves are influenced.

Alternatively, suppose we ask about the accuracy of judgments of color and of brightness and assume that the organism fluctuates in its ability to perform these tasks accurately, that is, that the organism has varying states which define its perceptual capability. Now we can ask whether there is a correlation between the state of the organism with regard to color judgments and with regard to brightness judgments, which is not to ask whether brightness is influenced by color but whether performance related to each of these attributes is correlated over time. Such a correlation does not require any process interaction.

Models of perceptual states are used most commonly, but not at all exclusively, in experimental situations in which thresholds are being measured. In such tasks, it seems reasonable that the basic sensitivities of the two eyes for example, or of two areas on the same retina, vary and that they vary in a correlated fashion. With such models, the correlation sought is between right and wrong responses with respect to the two or more channels, and in simplest fashion we simply ask whether errors are correlated. Models concerned with the independence of perceptual states can be considerably more sophisticated, however, allowing the assumption of several states, and with explicit statement of decision rules related to these states (see Eriksen, 1966, for an illustration).

The importance of the distinction for present purposes lies in the fact that assumptions about process independence necessarily enter into models concerned with perceptual states, and vice versa; and these relations can be fairly complicated. We will see, for example, that a particular type of process interaction can lead to performance on a joint perceptual task which is greater than parity, while at the same time producing a result with a state model which is less than parity. Perceptual processes and states of the organism are really part of the same total problem, and models of either require implicit or explicit assumptions about the other.

Logically, the distinction between process and state models can concern either the zero-

correlation or performance-parity criteria of independence. However, parity relations between process and state models are generally quite straightforward, so that there is little need to distinguish these two types of model with the parity criterion. Where the distinction is potentially very important is in regard to the correlation criterion, since process correlation can exist without state correlation and vice versa. Furthermore, the distinction between process and state becomes important when we have to be concerned with real or hypothetical decision rules, since decision rules can be applied with regard to either state or process.

This distinction between process and state independence, while discussed here in the specific context of perceptual independence, is not limited to immediate perceptual functioning at all but is a necessary distinction for any aspect of human performance. For example, two response systems may be non-independent in the sense that two direct processes interact, such as right- and left-hand simultaneous responding; or there may be correlated fluctuations in state produced by total organismic alertness, fatigue, etc. Similar distinctions need to be made with respect to memory processes. To illustrate, if both letters and numerals are held in memory, there could be direct process interaction between the classes of items, producing either interference or facilitation, but there could also be state fluctuations, either correlated for the two classes of items or independent, which affect the overall level of performance on the memory task. Thus the distinction is a very broad one applying to a wide range of psychological problems.

#### *Direct versus Indirect Models*

Nearly all models used in tests of perceptual independence can be classed as testing for independence either directly or indirectly. What constitutes directness is not so much a property of the model itself but rather is the relation of the model to the kind of independence being tested. Thus if zero correlation is being taken as the criterion of independence, a direct test of this criterion would require experimental measurement of correlation in some form. An indirect test would involve

measurement of some consequence of the lack of correlation. For example, suppose geometric patterns are presented to four different retinal locations, and separate identification of each is required. The number of correct identifications on a single trial can range from zero to four, and these should be distributed as the binomial if there is no correlation between performance at any of the retinal positions. This illustration is, of course, one of state correlation rather than process correlation, so the binominal test in this case is an indirect test of the zero correlation assumption in a state model (see Eriksen & Lappin, 1967, and Estes & Taylor, 1966, for examples).

The performance-parity criterion of independence can be tested indirectly as well. For example, if performance parity exists, then as we add perceptual tasks to an already existing one, performance on the first task should not decrease. If we take this lack of interference on one task as evidence of parity for both, we are testing the parity concept indirectly. This paradigm has been used especially in studies of memory.

We have already noted that performance parity can itself be considered as an indirect consequence of the more fundamental criterion of zero correlation. Yet because it can be and has been used as a direct criterion so often, it is better to consider models pertaining to performance parity as direct tests, and only those cases where indirect consequences of parity itself are involved should be considered as indirect tests.

### *Measurement Properties*

Another distinction concerning models is that of the type of measurement property assumed in the model. Since we are talking about mathematical models, the type of model which can be used depends very much on the kind of data which can be collected and the organismic processes which the data are presumed to reflect.

Models involved in the study of perceptual independence have involved essentially just three measurement properties:

1. Probability models, where simple class occupancy is involved. Models involving percentage correct responses, or binomial distribution, are of this type.

2. Information models, in which the information metric is applied to probability distributions. In one sense these models could be considered as a subclass of probability models. However, there are some sets of probability data to which the information metric cannot be meaningfully applied; furthermore, the information model is so intimately associated with the entire problem of perceptual independence (since the problem is really a question of independence of information processing) that it is useful to consider it as a special class of model.

3. Metric models, in which the variables are measured on some form of metric, usually involving at least interval scale properties, that is, the metric holds to a linear transformation. Models involving product-moment correlations, variances,  $d'$  (as in signal detection theory), and scales of perceptual distance are all of the metric type.

The measurement property of the model does not assume an important role as a logical problem, because the kind of metric used has much more to do with the specifics of the type of performance than with any general considerations involving type of independence and experimental paradigm. Any of the types of measurement can be used with respect to either criterion of independence, with process and state models, with direct or indirect models, and with any experimental paradigm. So the kind of measurement assumed in a model will enter into this discussion only as specific illustrations of points being made.

### EXPERIMENTAL PARADIGMS

There are many specific experimental arrangements which can be and have been used in the study of perceptual independence, depending to a great extent on the particular substantive topic under consideration. Here, however, since the concern is primarily with the general logic of such studies, four basic experimental paradigms are distinguished, and the distinctions depend entirely on the nature of the correlations between stimuli and between responses in the experiment. If perceptual independence is under investigation, then the independence involved in the stimuli themselves, and the independence in the responses required by the

experimental arrangement, must be crucial in our understanding of any independence which occurs between the input and the output of the organism. Clearly we cannot attribute independence or the lack of it to internal organismic processes unless we know the kinds of relations which exist within the stimuli and within the responses themselves.

### *Complete Orthogonal Experiments*

The basic, and in many ways ideal, experiment for investigating perceptual independence is one in which the stimuli themselves are orthogonal or uncorrelated, and in which the response arrangement is one which permits orthogonal response variables corresponding to the stimulus variables. That is to say, the stimulus variables are independent, and the response variables are capable of independence, so that any lack of independence in performance can reasonably be attributed to internal organismic limitations rather than to limitations inherent in the experimental arrangement itself.

*Process correlation.* This experimental paradigm is ideally suited to the use of models which use zero correlation as the criterion of independence and which are concerned with perceptual process. In this paradigm, there are manifest data in which correlation can exist, and if the correlation in the stimuli is known to be zero, and the correlation between responses could have been zero, then any obtained correlation may be attributed to properties of the organism.

Nevertheless, the actual measurement of correlations which indicate lack of perceptual independence is more complicated than at first glance it might seem. To illustrate, assume that we present a series of visual stimuli which vary in both area and brightness and that all areas occur in combination with all brightnesses; that is, the stimulus variables are themselves uncorrelated. Then we require the subject to make an identifying response, and on each stimulus presentation he must identify both area and brightness. The question is, given such data, what kinds of correlation can be measured which are meaningfully related to the problem of perceptual independence?

This problem is discussed with the information metric, because relations within such a set of data have been well worked out (see Garner, 1962). Furthermore, the information metric provides a closed analytic system, one in which we can identify the total constraint existing in a system of variables and then can partition this constraint into various meaningful terms while at the same time being assured that no part of the total constraint has been left out. Thus it is the logical properties of the information metric which are important, rather than the measurement properties per se. Once the various terms have been identified, any of a number of measures of performance or correlation can be used with equal effectiveness.

There are four variables to consider: Two stimulus variables, area ( $A$ ) and brightness ( $B$ ), and two response variables, response to area ( $a$ ) and response to brightness ( $b$ ). We can write an expression for the total constraint within this system of four variables,  $U(a:b:A:B)$ , and this total constraint can then be completely partitioned into several components especially meaningful for the question of perceptual independence:<sup>3</sup>

$$U(a:b:A:B) = U(A:B) + U(a:A) + U(b:B) + U_B(b:A) + U_A(a:B) + U_{AB}(a:b). \quad [3]$$

The term  $U(A:B)$  is the contingency (or non-metric correlation) between the two stimulus variables, and it is zero because we use these variables orthogonally. The next two terms are the direct contingencies in which each response variable is correlated with its appropriate stimulus variable. These are the terms we would expect to be high, since they indicate the extent to which the subject can

<sup>3</sup> This particular partition of the total constraint has not been used before. Its derivation can be seen from equations given by Garner (1962). From Garner's Equation 5.8,

$$U(a:b:A:B) = U(a:b,A,B) + U(b:A:B).$$

From Equation 4.11, the first term on the right becomes

$$U(a:b,A,B) = U(a:A) + U_A(a:B) + U_{AB}(a:b).$$

From Equations 5.3 and 4.4, the second term on the right becomes

$$U(b:A:B) = U(A:B) + U(b:B) + U_B(b:A).$$

Combining these terms gives Equation 3 in the present paper.



accurately judge area and brightness. And the normative model for perceptual independence is that only these two terms are nonzero.

The last three terms all have something to do with the question of perceptual independence. Two of the terms are of the same form,  $U_B(b:A)$  and  $U_A(a:B)$ . These are partial cross contingencies (exactly analogous to partial correlation), each of them measuring the correlation between a response variable and the inappropriate stimulus variable, with the effect of the appropriate stimulus variable partialled out. These two terms are directly relevant to the question of perceptual independence, since each of them describes a sort of "crossing over" from one perceptual system to another. To illustrate, suppose that judged brightness is increased with large areas and that judged area is increased with high brightness; the effect would be that the response to area is correlated with stimulus brightness, and the response to brightness is correlated with stimulus area. Thus the cross contingencies are greater than zero.

The last term in Equation 3 is the partial contingency between the two response variables with both stimulus variables held constant, their effects thus being partialled out. At first glance it would appear that such a term is not concerned with *perceptual* independence, since it measures response correlation with stimulus effects partialled out. Yet as seen in more detail subsequently, such a term indicates error correlation, which can clearly be perceptual error as well as response error. Furthermore, this term, while being a

TABLE 1  
COMPLETE INDEPENDENCE

Complete response variable	Orthogonal stimulus variable				$\Sigma$
	$A_1B_1$	$A_1B_2$	$A_2B_1$	$A_2B_2$	
$a_1b_1$	9	6	6	4	25
$a_1b_2$	6	9	4	6	25
$a_2b_1$	6	4	9	6	25
$a_2b_2$	4	6	6	9	25
$\Sigma$	25	25	25	25	100

Note.—Data matrices are for the case in which stimulus variables  $A$  and  $B$  are each responded to with 60% accuracy, and both perceptual processes and response processes operate completely independently. Cell entry is percentage of joint occurrence of multiply defined stimulus and multiply defined response.

TABLE 2  
DATA FROM TABLE 1 REDUCED TO  
PERCENTAGE CORRECT

Variable	Variable		$\Sigma$
	$A +$	$A -$	
$B +$	36	24	60
$B -$	24	16	40
$\Sigma$	60	40	100

Note.—"+" is correct and "-" is incorrect. Cell entry is percentage of joint occurrence of correctness for stimulus variables  $A$  and  $B$ . Tables 1-10 are all of the same form and the cell entries have the same meaning as indicated here.  $U(A_2:B_2) = 0$ . Percentage correct for: inclusive disjunction = 84; 50-50 on doubtfuls = 60; conjunction = 36.

measure of process correlation, is directly related to state correlation.

In order to clarify the uncertainty analysis and in order to give some additional meaning to the terms which are relevant to the problem of perceptual independence, the authors have prepared five sets of idealized data which illustrate conceptually different types of process correlation. These data, in the complete orthogonal form, are given in Tables 1-10; the actual data matrices are given in the odd-numbered tables, and the reduced data matrices in the even-numbered tables. Uncertainty calculations from each set of data are shown in Table 11. In each case, dichotomized stimulus variables  $A$  and  $B$  and dichotomized response variables  $a$  and  $b$  (the subscripts 1 and 2 indicate the level of each stimulus or response variable) are used. Also in each case, each level of each stimulus variable always occurs equally often and the two variables are orthogonal in keeping with the immediate concern. In addition, each level of each response variable occurs equally often, although in three cases there is some correlation between responses.

*Complete independence* is illustrated in Tables 1 and 2, and this case is the normative model for perceptual independence. Stimulus variable  $A$  is responded to with response variable  $a$  with 60% accuracy, and there is equivalent accuracy for  $B$ . These two terms show in Table 11 as 0.029 bit of direct contingent uncertainty each, and the total constraint in the system of four variables is 0.058, the sum of these two terms. There is no cross correlation (i.e.,  $b$  is not correlated with  $A$ ,

TABLE 3  
ERROR CORRELATION

Complete response variable	Orthogonal stimulus variable				$\Sigma$
	$A_1B_1$	$A_1B_2$	$A_2B_1$	$A_2B_2$	
$a_1b_1$	11	4	4	6	25
$a_1b_2$	4	11	6	4	25
$a_2b_1$	4	6	11	4	25
$a_2b_2$	6	4	4	11	25
$\Sigma$	25	25	25	25	100

Note.—Data matrices are for the case in which stimulus variables  $A$  and  $B$  are each responded to with 60% accuracy, and there is a tendency for both response variables ( $a$  and  $b$ ) to be right together or wrong together; that is, errors are correlated.

nor  $a$  with  $B$ ), and there is no correlation between the responses, that is, there is no error correlation.

*Error correlation* is illustrated in Tables 3 and 4, there being perceptual independence otherwise. Stimulus variables  $A$  and  $B$  are still each responded to with 60% accuracy, thus giving 0.029 bit of direct contingent uncertainty in Table 11. However, if  $a$  is in error to  $A$ , the probability that  $b$  is in error to  $B$  is increased. In Table 11 this relation shows up as the double partial term,  $U_{AB}(a:b)$ , with a value of 0.081 in the actual illustration. There is, however, no cross correlation, each response variable responding only to its appropriate stimulus variable. The total constraint in this case is 0.139, which is the constraint for the complete independence case plus the additional constraint in the response system. Note that response or error correlation produced this way does not produce an overall contingency between the two response variables:  $U(a:b)$  is zero. Thus it is important to differentiate between the simple

TABLE 5  
JOINT RESPONSE BIAS

Complete response variable	Orthogonal stimulus variable				$\Sigma$
	$A_1B_1$	$A_1B_2$	$A_2B_1$	$A_2B_2$	
$a_1b_1$	11	8	8	6	33
$a_1b_2$	4	7	2	4	17
$a_2b_1$	4	2	7	4	17
$a_2b_2$	6	8	8	11	33
$\Sigma$	25	25	25	25	100

Note.—Data matrices are for the case in which stimulus variables  $A$  and  $B$  are each responded to with 60% accuracy, but there is a joint response bias such that the same  $a$  and  $b$  responses are made more frequently than different  $a$  and  $b$  responses.

contingency between responses and the contingency with the stimulus effects partialled out.

*Joint response bias* is illustrated in Tables 5 and 6 and is a different way in which there can be constraint in the response system. (Once again, the two direct contingencies each give 0.029 bit of contingent uncertainty, indicating the same level of 60% accuracy for each stimulus variable.) In Tables 3 and 4 the correlation was between errors in the two systems, but the proportion of times that each combination of responses was made remained at 0.25. In Tables 5 and 6, however, all response combinations are not used equally often, even though both levels of each response variable are used equally often. This is to say that there is correlation between the responses, measured by  $U(a:b)$  in Tables 5 and 6. A simple case where this might happen is in a threshold test for two visual stimuli at two retinal locations. If the subject reports seeing both or neither more often than one and not the other, he is exhibiting a joint

TABLE 4  
DATA FROM TABLE 3 REDUCED TO  
PERCENTAGE CORRECT

Variable	Variable		$\Sigma$
	$A+$	$A-$	
$B+$	44	16	60
$B-$	16	24	40
$\Sigma$	60	40	100

Note.— $U(A_c:B_c) = 0.081$ . Percentage correct for: inclusive disjunction = 76; 50-50 on doubtfuls = 60; conjunction = 44.

TABLE 6  
DATA FROM TABLE 5 REDUCED TO  
PERCENTAGE CORRECT

Variable	Variable		$\Sigma$
	$A+$	$A-$	
$B+$	36	24	60
$B-$	24	16	40
$\Sigma$	60	40	100

Note.— $U(A_c:B_c) = 0$ . Percentage correct for: inclusive disjunction = 84; 50-50 on doubtfuls = 60; conjunction = 36.

TABLE 7  
SINGLE PERCEPTUAL PROCESS

Complete response variable	Orthogonal stimulus variable				$\Sigma$
	$A_1B_1$	$A_1B_2$	$A_2B_1$	$A_2B_2$	
$a_1b_1$	9	9	4	4	26
$a_1b_2$	6	6	6	6	24
$a_2b_1$	6	6	6	6	24
$a_2b_2$	4	4	9	9	26
$\Sigma$	25	25	25	25	100

Note.—Data matrices are for the case in which both response variables  $a$  and  $b$  are in response to stimulus variable  $A$ , each with 60% accuracy, and the two responses are otherwise made independently of each other.

response bias. The consequences of such a bias, as seen in Table 11, is also to produce a nonzero value for  $U_{AB}(a:b)$ , and a total constraint which is increased from the independence case by this amount. So unlike Tables 3 and 4, in this case there is contingency between responses both with and without the effects of the stimulus partialled out.

A *single perceptual process* is illustrated in Tables 7 and 8 but with independent response processes. For this case it is assumed that both response variables,  $a$  and  $b$ , are made in response to stimulus variable  $A$ , each with 60% accuracy, and that neither is correlated with stimulus variable  $B$ . However, beyond this fact, the two response variables are made independently of each other insofar as possible. In uncertainty terms, the direct contingency,  $U(a:A)$ , remains at 0.029, but instead of the other direct contingency term having this same value, it now appears as the partial cross contingency,  $U_B(b:A)$ , with the same value of 0.029. And the total constraint is the same as it was for the independence case

TABLE 8  
DATA FROM TABLE 7 REDUCED TO  
PERCENTAGE CORRECT

Variable	Variable		$\Sigma$
	$A +$	$A -$	
$B +$	30	20	50
$B -$	30	20	50
$\Sigma$	60	40	100

Note.— $U(A_e:B_e) = 0$ . Percentage correct for: inclusive disjunction = 80; 50-50 on doubtfuls = 55; conjunction = 30.

TABLE 9  
SINGLE RESPONSE PROCESS

Complete response variable	Orthogonal stimulus variable				$\Sigma$
	$A_1B_1$	$A_1B_2$	$A_2B_1$	$A_2B_2$	
$a_1b_1$	15	15	10	10	50
$a_1b_2$	0	0	0	0	0
$a_2b_1$	0	0	0	0	0
$a_2b_2$	10	10	15	15	50
$\Sigma$	25	25	25	25	100

Note.—Data matrices are for the case in which response variable  $a$  is used with 60% accuracy in response to stimulus variable  $A$ , and response variable  $b$  is then copied from  $a$ ; that is, the two response variables are perfectly correlated and are used in response to  $A$  with 60% accuracy.

in Tables 1 and 2. While this arrangement produces no error correlation,  $U_{AB}(a:b)$ , there is a small amount of contingency between the two responses when the effect of the stimulus is not partialled out,  $U(a:b)$ , and this is due to the fact that two things each correlated with a third must be correlated with each other, the extent depending on the degree of correlation each has with the third.

A *single response process* is illustrated in Tables 9 and 10. The two response variables are perfectly correlated, and then this effectively unitary response is made only to stimulus variable  $A$  and with 60% accuracy. So as with a single perceptual process, there is 0.029 bit of direct contingent uncertainty plus an equal amount of partial cross contingent uncertainty. In addition, however, there is now a large error correlation,  $U_{AB}(a:b)$  being 0.971 bit. This term also enters into the total constraint, which now becomes 1.029 bits. Note that the direct interresponse contingency,  $U(a:b)$ , is 1.000 bit, which is simply a

TABLE 10  
DATA FROM TABLE 9 REDUCED TO  
PERCENTAGE CORRECT

Variable	Variable		$\Sigma$
	$A +$	$A -$	
$B +$	30	20	50
$B -$	30	20	50
$\Sigma$	60	40	100

Note.— $U(A_e:B_e) = 0$ . Percentage correct for: inclusive disjunction = 80; 50-50 on doubtfuls = 55; conjunction = 30.

TABLE 11  
UNCERTAINTY ANALYSES FOR THE COMPLETE  
MATRICES OF TABLES 1-10

Uncertainty term	Data matrix				
	1	3	5	7	9
$U(A:B)$	0	0	0	0	0
$+U(a:A)$	0.029	0.029	0.029	0.029	0.029
$+U(b:B)$	0.029	0.029	0.029	0	0
$+U_B(b:A)$	0	0	0	0.029	0.029
$+U_A(a:B)$	0	0	0	0	0
$+U_{AB}(a:b)$	0	0.081	0.082	0	0.971
$=U(a:b:A:B)$	0.058	0.139	0.140	0.058	1.029
$-U(a:b)$	0	0	0.075	0.001	1.000
$=U(a,b:A,B)$	0.058	0.139	0.065	0.057	0.029

Note.—The cell entry is bits of contingent uncertainty. The symbols to the left of the notations for the uncertainty terms indicate the mathematical relations between the terms (see Equations 3 and 7).

description of our statement that the two response variables are perfectly correlated.

Thus we have illustrated the normative model for complete perceptual independence, plus four cases of nonindependence, each of these four being based on a different conceptualization of how the correlations within and between the two systems occur. Each produces an identifiably different pattern of uncertainties when a complete analysis is carried out. Of the three uncertainty terms identified with process correlation, it might seem that the partial response contingency,  $U_{AB}(a:b)$ , is possibly not really related to *perceptual* independence, but that term is intimately related to perceptual correlation in state models.

*State correlation.* As we noted above, state models are not concerned with direct interaction of the perceptual processes themselves but rather with the state of the organism in respect to the two or more channels of information being processed. The usual experimental question is not whether the processes are correlated but whether accuracies are correlated.

This problem is handled on a simple level here, using the same idealized data presented in Tables 1, 3, 5, 7, and 9, and asking about the relation between data put in form to test state correlation to data in the original form. To accomplish this, each of these five sets of hypothetical data has been reduced

to a  $2 \times 2$  matrix, in which the entry is the joint percentage of times that  $A$  and  $B$ , the stimulus variables, were responded to correctly. To illustrate the relation between the reduced and original matrices, consider Tables 1 and 2. The entry for  $A+B+$  in the reduced matrix is the sum of the diagonal entries for  $A_1B_1a_1b_1$  plus  $A_1B_2a_1b_2$ , etc. The entry for  $A-B-$  is the sum of the other diagonal, indicating that errors are made with respect to both stimulus variables. The entry for  $A+B-$  in the reduced matrix is the sum of the terms in which  $A$  was correctly responded to and  $B$  was incorrectly responded to, for example,  $A_1B_1a_1b_2$  plus  $A_1B_2a_1b_1$ , etc. The last entry in Table 2 is computed in like fashion.

These reduced matrices are shown for each of the five cases. In the note to the reduced matrix is also shown another contingent uncertainty, indicated as  $U(A_c:B_c)$ , which is the contingent uncertainty or correlation between correctness of response on the two variables. It is the correlation term directly related to state correlation. The other statistics shown there relate to decision rules and are discussed later.

First, note the relations between the two matrices for each set of data. The reduced matrix has marginal terms which indicate the average performance level for each of the two stimulus variables. We score correct only in relation to the appropriate response variables, so these marginal terms indicate the same thing as the direct contingency terms do in the complete orthogonal matrix. All of the matrices are idealized, so the probability of being correct is always independent of the level of the stimulus variable. Thus to combine these levels into a single measure of right or wrong does not lose any information. If the probability of being right itself depended on the level of the stimulus variable, this information would have been lost in the reduced matrix.

But now consider each of the five reduced matrices, those which are to tell us about state correlation. In only one case, Table 4, is the appropriate contingent uncertainty,  $U(A_c:B_c)$ , nonzero. Thus we would conclude that there is perceptual independence in four of these five cases, even though the original data

were set up so that four out of the five cases were clearly indicative of perceptual non-independence or correlation. In Table 4, the contingent uncertainty between correctness on  $A$  and on  $B$  is in fact the same as the partial contingent uncertainty between responses with stimuli held constant in the orthogonal matrix,  $U_{AB}(a:b)$ . That case was described as one in which there was error correlation, which is after all what should be happening if there is state correlation. Note, however, that the existence of such a nonzero term in the orthogonal matrix does not necessarily mean that there will be correlation in the matrix reduced to investigate state correlation, since there is also such a term in Tables 5 and 9, and neither of them shows state correlation in Tables 6 and 10.

The relationship between the values of the uncertainty terms in the full matrix and those in the reduced matrix is complex. To some extent this complexity is due to the fact that the reduced matrix is obtained by, in effect, arithmetic averaging of terms in the full matrix, while the uncertainty terms themselves are based on logarithmic equivalents of the various proportions. In addition, however, the relations themselves are complex, so that, for example, it is possible to have both correlated error and a joint response bias, in which case  $U_{AB}(a:b)$ ,  $U(a:b)$ , and  $U(A_c:B_c)$  will all be nonzero. The usefulness of the reduced matrix must then lie in the possibility of distinguishing between the case where there is joint response bias alone and the case where this is accompanied by error correlation. What is certainly clear is that analysis of the reduced matrix by itself yields very little information.

Still further, the existence of a correlation in the reduced form does not by itself differentiate perceptual process from response process, since such a term indicates only that errors are correlated, and they could be correlated as a consequence of perceptual correlation or of response correlation. Neither does the analysis from the orthogonal matrix make such a differentiation. Nevertheless, a far more complete and informative picture of process can be obtained from complete analy-

sis of the orthogonal matrix than from the reduced matrix.

*Data reduction and decision rules.* As pointed out previously, models may test for perceptual independence indirectly as well as directly. In the present case, an easily used indirect test would be simply to count the total number of correct responses rather than to measure directly correlation in the reduced matrix. However, before we can carry out such a procedure, we (the experimenters, that is) have to establish some decision rule to determine what constitutes being correct, since two of the four cells in the reduced matrix are cases where the subject is correct with respect to one attribute and incorrect with respect to the other.

There are three prototypical rules for handling these cases of partial correctness, and the percentage correct computed for each of these rules is shown in the note to each of the five reduced matrices:

1. *Inclusive disjunction:* Credit for being correct is given if the subject is correct on either or both stimulus variables. This is the rule that leads to the familiar normative model value of  $(p_A + p_B - p_A p_B)$ , where  $p_A$  and  $p_B$  are the respective probabilities of being correct on the two stimulus variables.

2. *50-50 on doubtfuls:* When the subject is correct on one variable and incorrect on the other, give him credit for being correct on half the total number of such cases, plus those cases in which he is correct on both.

3. *Conjunction:* Credit for being correct is given only when the subject is correct on both.

It is of interest to compare the percentage of correct responses which would be obtained for each of the five idealized matrices, given the actual marginal values and correlations shown. Table 2, representing the case of complete independence, provides the base values to be expected with each decision rule.

Two of the matrices, Tables 8 and 10, represent cases where only chance correctness occurs with respect to the  $B$  variable, and where there is no state correlation. Lowered performance occurs for these two matrices for any of the three decision rules, and this lowered performance simply reflects the lack of accuracy with one stimulus variable.

Each of the other two matrices has equal performance on stimulus variables  $A$  and  $B$  separately. Table 6 also has zero state correlation, and thus each decision rule gives the same performance as expected from the normative Table 2. Table 4 has state correlation, and this correlation produces percentage correct values for two of the decision rules which are different from those obtained in Table 2, but these two rules give differences from the independence case in opposite directions. With the inclusive disjunction rule, there is lower performance with correlated accuracy than with complete independence, while with the conjunction rule, performance is better with state correlation than without it.

Just as it is argued that reduced data matrices should not be used to test directly for state correlation if the orthogonal matrix is available, here it must be argued that indirect tests of state correlation by measurement of percentage correct is not reasonable. Not only does it confound performance on the separate variables with the correlation problem, but it further introduces the complication of having to use a decision rule; and unfortunately the decision rule chosen affects apparent performance differentially for correlated and uncorrelated cases. Thus in general there is little justification for using any reduced form of data for testing perceptual independence if it is possible to obtain and use data in the more complete form.

#### *Orthogonal Input—Unitary Response*

Up to this point, the paper has been considering the experimental paradigm wherein the stimulus variables are orthogonal and the subject may use his response variables orthogonally, whether or not he does so. Now, models of independence when the experimental situation presents orthogonal stimulus variables, but the subject must respond with a single output variable, must be considered. There are two paradigms within which such a requirement would be reasonable:

1. The nature of the stimulus variables makes some sort of composite response reasonable. For example, the experimenters might

carry out a threshold experiment with lights presented or not presented, in all possible combinations, to the two eyes separately but simultaneously. The subject is then required only to state whether he did or did not see a light. Alternatively, stimuli varying in area and brightness could be used, but with the subject being required only to give a single response, perhaps indicating a kind of composite or total magnitude.

2. The subject is instructed to respond selectively to one stimulus channel or variable alone, even though two orthogonal channels of information are used as the stimulus. (In effect this is what the subject has done for himself with the idealized data of Tables 9 and 10). Ordinarily such a procedure would be used when the experimenters are thinking in terms of interference effects (i.e., the second channel is a distractor, is an irrelevant variable, etc.).

*Process correlation.* When measuring process correlation in these two cases, the composite response or selective response are the same. In either case this is an undifferentiated response variable, symbolized as  $R$ , the difference between the two cases being in instructions to the subject rather than the form of the data.

For a direct test of process correlation, the informational analysis is much simpler than before, since only three terms, the two stimulus variables (still  $A$  and  $B$ ) and one response variable ( $R$ ) are involved. The total constraint in this system of three variables,  $U(R:A:B)$ , can be partitioned into four terms:

$$U(R:A:B) = U(R:A) + U(R:B) + U(\overline{R}A\overline{B}). \quad [4]$$

The first term on the right is again the contingent uncertainty between the two stimulus variables and is zero because the stimulus variables are still orthogonal. Each of the next two terms is the direct contingent uncertainty which indicates the extent to which the single response variable is correlated with the stimulus variable. The last term is an interaction uncertainty and is the only term directly relevant to the question of perceptual

independence. Its relation to this question can be seen by writing

$$U(\overline{RAB}) = U_A(R:B) - U(R:B) \quad [5]$$

$$= U_B(R:A) - U(R:A) \quad [6]$$

Thus the interaction term is a difference between the contingent uncertainty relating the response and one of the stimulus variables and that same term with the other stimulus variable held constant, its effect being thus partialled out. If the relation between the response and a stimulus variable is changed by holding the other stimulus variable constant, then there is an effect of that second stimulus variable on the percept pertinent to the first variable. Note, however, that since both Equations 5 and 6 are true, the effect cannot be differentiated as being due to *A* or to *B* without further experimental controls. In like manner, the cross contingencies cannot be identified, since with an undifferentiated response it is not possible to tell how much of the correlation of the response with a stimulus variable is appropriate and how much is inappropriate.

Thus the use of a single response in an experiment, even though at times appropriate to a particular hypothesis, produces a drastic reduction in our ability to determine the amount and nature of perceptual process correlation.

*State correlation.* There is no reasonable way in which a direct test can be made for state correlation with unitary response data. Since the subject does not separately identify his response to *A* and his response to *B*, the experimenter in turn cannot separately identify when he is correct on *A* but not on *B*, etc. Thus data matrices similar to the reduced matrices in Tables 2, 4, 6, 8, and 10 cannot be generated directly from the available data. This fact means that the only possible test of state correlation must be indirect.

Given experimental questions for which the use of a unitary response is appropriate, it is rare that a test of state correlation seems necessary or desirable. If the subject is directed to attend and respond to only one channel or variable, the experimental interest is normally in parity not in correlation. And if

it is meaningful to make a composite response, the interest is more with the actual response than with whether it is right or wrong. There is one type of situation in which the concept of state correlation does make sense, and in which a unitary response also makes sense, and that is the threshold case where two stimuli are presented, all combinations of presence or absence being used, and the subject simply responds that he did or did not perceive a stimulus. Now there is a criterion by which correctness can be decided; namely, if either stimulus was presented, the subject should have responded positively.

An indirect test would then compare the overall percentage correct responses made against a normative model which assumes zero correlation between the states for the two separate stimuli. A hypothetical matrix such as the reduced matrices of Tables 2, 4, 6, 8, and 10 would be generated, and from them a total percentage correct figure would be obtained. However, with a single experiment the indirect test still could not be made because there is no way of determining percentage correct responses for each stimulus separately, and these are required in order to generate the hypothetical matrix with zero correlation. Thus no meaningful test of state correlation can be made with this experimental paradigm alone.

*Parity and orthogonal inputs.* In the experimental situation in which orthogonal inputs are used, and especially when orthogonal responding is permitted, the data make direct calculation of correlations possible, and thus are ideally suited to direct testing of the correlation criterion. Even when data are reduced to percentage correct form, state correlation can be directly calculated. The parity criterion of independence is simply not applicable with this experimental paradigm, because any parity check is simply an indirect test of the correlation criterion of independence. Thus performance parity with this paradigm has no meaning separate from that of zero correlation. It is worth elaborating this problem a bit more.

If informational analysis is used again, a multiple contingent uncertainty indicates total performance between the two stimulus vari-

ables and the two corresponding response variables. It is written as  $U(a,b:A,B)$  and is related to the total constraint in the system of four variables as

$$U(a,b:A,B) = U(a:b:A:B) - U(a:b) - U(A:B). \quad [7]$$

Thus total performance in which a multiple response is contingent on a multiple stimulus equals the total constraint in the system minus the constraint within the system of response variables themselves and minus the constraint within the system of stimulus variables. Since the case of orthogonal input is being discussed, then the latter term,  $U(A:B)$ , is zero by definition. Thus the only term which produces a difference between the total constraint and the multiple contingent uncertainty is the simple contingent uncertainty,  $U(a:b)$ , showing the extent to which the response variables are correlated.

The apparent performance-parity check would be to compare the magnitude of the multiple contingent uncertainty,  $U(a,b:A,B)$ , with the sum of the two direct contingencies,  $U(a:A) + U(b:B)$ . A brief comparison for each of the five data matrices, as shown in Table 11, will clarify the fact that a discrepancy between these two measures shows only that there is correlation somewhere in the system, although it does not locate the source of the correlation.

For Table 1, the case of independent processing, the sum equals the multiple term, which indicates that all correlations other than the direct contingencies are zero. For Table 3, the sum is substantially smaller than the multiple contingent uncertainty. Thus parity is not satisfied, and it is known that correlation exists but not where. In Table 5, there is also a discrepancy, although comparison of this result with that of Table 3 shows the importance of complete analyses. These two matrices look alike with respect to total constraint but quite different with respect to total in-out performance. For Table 7, the multiple contingent uncertainty is greater than the sum of the two direct contingencies (one of them being zero), so once again parity is not satisfied; therefore, there is correlation somewhere in the total system.

Table 9 is the most interesting, because with it parity is satisfied, the multiple contingent uncertainty being the sum of the two direct contingencies. Thus, one would conclude that the processes are independent, when in fact this matrix shows data with the greatest degree of perceptual and response interdependence.

Thus the use of parity checks with data in which correlation can be computed directly serves only as an indirect measure of what can be measured directly; furthermore, it serves this purpose very inadequately, failing to locate the source of correlation when correlation is indicated, and in some cases even failing to indicate the existence of correlation.

This limitation in the use of parity checks with single experiments, because parity is defined in terms of zero correlation, is in no way restricted to the use of the information model. All metric models have exactly the same limitation, as shown basically in Equation 1, where parity is defined in terms of zero correlation. If we use  $d'$  measures (see Green & Swets, 1966; Tanner, 1956), or some of the dimensional scaling procedures (see Torgerson, 1958), then parity is defined with the Pythagorean relation, so that, for example,

$$d'^2_{AB} = d'^2_A + d'^2_B. \quad [8]$$

This relation, however, holds only if the two processes are uncorrelated. Usually this relation is said to constitute a Euclidean geometry, although the primary consideration is this Pythagorean relation (Euclid's proposition I:47), and this relation holds (as every schoolboy knows) only for right triangles, that is, orthogons, from which the general term orthogonality for zero correlations comes. If we do an experiment and find that parity does not hold, then we can assume that there was a correlation, that is, that the relationship was not of a right triangle. Alternatively, of course, one could assume that the model is wrong. On the other hand, if parity appears to hold, the result could be due to a correlation which offsets a true loss of parity.

The limitations of the single orthogonal experiment in determining whether parity exists are strongest with a linear correlation model used with a unitary response system. If



linear correlations are computed, then the squared multiple correlation coefficient is

$$R^2_{R:A,B} = r^2_{R:A} + r^2_{R:B}. \quad [9]$$

(For the correlations, subscript notation has been used like that used with uncertainty measures to emphasize the parallel relations. The subscripted  $R$  is the unitary response variable.) This equation, again showing the Pythagorean relation so common in statistics which use variance measures, cannot be wrong. The parity relation holds if the predictor variables are uncorrelated, and in this experimental paradigm they are. So if we measure each correlation separately, then measure the multiple correlation directly, and find that the sum of the two squared separate correlations equals the square of the multiple correlation, we have learned exactly one thing, namely, that we have done our arithmetic properly. Thus with this particular model and the experimental paradigm of an orthogonal input but unitary response, we can ask no meaningful question about perceptual independence.

#### *Single Input Experiments as Controls*

Since the single experiment with orthogonal input channels is not satisfactory in providing a check on the performance-parity criterion of independence, some separate measure of performance is needed outside of the orthogonal experiment itself. The most natural technique is to run additional control experiments in which one channel or variable is used at a time, with responses appropriate to that single channel or variable. There is little complexity about handling the data in this case, since they would be handled in the form in which the parity comparison would be made; for example, either correlations or contingencies as measures of performance, or percentage correct responses as in the reduced matrix, can be used as long as the measures correspond in the orthogonal and single input experiments.

The parity checks themselves are quite straightforward in most cases and involve comparison of the performance measure obtained in the single input experiment with the most analogous term obtained in the larger

orthogonal experiment. When an orthogonal response system has been used, then (using the information measures) there will be two direct contingencies from the orthogonal experiment and two simple contingencies from the single channel experiments, one for each variable or input. If performance measures are larger with the single input experiment than with the orthogonal experiment (or possibly smaller), then parity is not satisfied.

If a unitary response has been used with the orthogonal experiment, then there still are appropriate direct contingencies available to compare to the simple contingencies obtained in the single input experiments. However, if the unitary response is a selective response to just one stimulus input, then there would ordinarily be just the one comparison. This type of comparison, of course, is ideally suitable when interchannel interference effects are expected.

There might be some question about what the proper comparison is in making the parity check. We have suggested that the meaningful comparison is to compare separately equivalent terms from the two experiments. An alternative that might be considered is to compare total performance from the orthogonal experiment, the multiple contingent uncertainty  $U(a,b:A,B)$ , with the sum of the two simple contingencies obtained in the two control experiments. Such a comparison is not valid, however, since the total performance measure, as discussed above and as shown in Table 11, contains process correlation terms as well as direct performance terms. To illustrate, Table 3 would give a total performance greater than the sum of the two simple contingencies (assuming them to be the same as the two equivalent direct contingencies in Table 11), but this greater performance is due entirely to error correlation. Certainly an independent criterion of performance parity does not want to be concerned with such terms.

While it is possible to make indirect checks of performance parity, by observing some consequences of the parity assumption rather than the performance itself, it is unusual to find an example where an indirect check does not confound the two kinds of independence.

The essential dependence of the parity criterion on correlation was noted previously. As one example of a more indirect test, the binominal distribution of "hits" might be determined on the basis of probabilities obtained from the single-channel control experiments. If the expected distribution does not agree with the obtained distribution, the discrepancy could be due either to a loss of parity or to the existence of correlation. If direct comparison were then made of the overall hit rate in the orthogonal experiment with the average hit rate from the single channel experiments, the difference would give information about parity, and the shape of the distribution would then give information about correlation. But this separation means we have gone to a direct test of parity, leaving only correlation as an indirect test.

The performance-parity criterion of perceptual independence, and one which is quite separable from the correlation criterion, is especially necessary when we consider the large number of experiments in which it simply is not feasible to measure correlation in any direct fashion. All of the experiments which use time, either reaction time or speed, as the measure of performance fall into this category. Performance as correlation with an input variable cannot be measured in these experiments, because maximum accuracy is expected of the subject, with differences in time then presumed to indicate differences in difficulty or performance. Also, those experiments in which the stimulus inputs do not have measurable correlation (e.g., separate speech channels) need to use the performance-parity criterion of independence. Even though the formal logic of the relation between parity and correlation is not applicable in these cases, the conceptual problem of untangling these two aspects still exists.

*Other control conditions.* There are control experiments other than the use of a single input channel that are useful in determining the existence or nature of perceptual independence. One ideal set of experiments might be worth mentioning because it forms a fairly complete and self-contained set of excellent converging operations (Garner, Hake, & Erikson, 1956). There would be, for a two-variable

stimulus such as visual area and brightness, the complete orthogonal experiment. Then two control experiments would be run in which each stimulus variable is used alone. Then two more control experiments would be run using the orthogonal stimulus set, but with the subject being required to respond selectively to just one stimulus variable in each experiment. Direct calculation of all contingent uncertainty terms would be made from the orthogonal experiment. Then the two obtained direct contingency terms would be compared against the values obtained with the single input experiments. Suppose these values are lower with the orthogonal experiment, so that there is a loss of parity. Now compare each of these simple contingencies from the single input experiment to the values obtained when orthogonal stimuli with selective responding were used. If the comparisons show no difference, the conclusion would be that additional perceptual load per se does not lead to lower performance, so that the lowered performance obtained with the complete orthogonal experiment is due to the extra task requirement of responding to both variables. Alternatively, a result showing that the loss occurs when the stimulus variables are orthogonal, regardless of whether response must be made to one variable or both, would indicate that the parity loss is almost certainly due to some perceptual incapacity. Still further, both the complete orthogonal experiment and the selective response experiments have terms which can be identified as cross contingencies. Comparison of these terms would lead to clarification of the nature of the process correlation.

In other words, to gain a reasonably accurate picture of whether and what kind of perceptual independence exists requires not one but several experiments all hopefully converging on the same presumed process. (As an illustration using  $d'$  measures, see Taylor, Lindsay, and Forbes, 1967.)

### *Correlated Input Experiments*

The last major experimental paradigm considered is that in which the stimulus variables, inputs, or sources of information are themselves correlated. For example, if geometric

forms are to be discriminated, the form is always the same regardless of the number of them. Or if speech context is one source of information, and tachistoscopic presentation is another form, the context is appropriate to the word tachistoscopically presented. Still further, if judgments of visual area and brightness are required, a given brightness is paired with a given area, and only a correlated subset of the orthogonal combinations is used.

From the viewpoint of understanding perceptual independence, the widespread use of this experimental paradigm is unfortunate, because learning something about independence of the perceptual process is fundamentally easier if the stimulus variables are themselves independent. The use of correlated stimuli confounds the problem of independence and also provides such a reduced set of data that very limited information about perceptual independence is obtainable from such experiments.

Yet this experimental paradigm must be considered, and we must attempt to understand it, because the paradigm represents a problem in and of itself. It is the problem of the organism's ability to make use of redundant information, and some of the earliest experiments done on the problem were concerned with this very problem (e.g., Eriksen & Hake, 1955). The question of perceptual independence enters in because the ability of the organism to use redundant information obviously depends on the extent to which this externally correlated information is processed independently internally, and Garner and Lee (1962) argued that there should be no gain in perceptual performance with redundant stimulus information unless the perceptual processes are independent. So the problem of perceptual independence is a critical aspect of the problem of utilization of redundant information.

A major problem in studying perceptual independence with correlated inputs is that the data are necessarily so reduced. We have seen that quite detailed analysis and understanding of independence are possible if both stimuli and possible responses are orthogonal and that the analysis becomes quite restricted if a unitary response system is used. Yet, as

Equation 4 shows, some direct measurement of process correlation is possible with a unitary response if the stimuli are orthogonal, although there is no reasonable way of directly measuring state correlation. If in addition to limiting the response we now also limit the input by correlating stimulus values, we completely destroy any possibility of direct measurement of either process or state correlation because we have only a single, two-dimensional, input-output matrix of data available. Therefore we may use only indirect models which state some consequence of the parity and/or correlation criteria of independence, and if correlation, then with respect to process and/or state correlation. Further, as elaborated later, decision rules need to be assumed, and these rules likewise can be specified with regard to perceptual process or perceptual state. Thus an indirect normative model which states what kind of performance should be obtained with correlated input must deal with more conceptual distinctions than our models can ordinarily handle.

*Parity and/or correlation.* Performance with correlated stimuli will be affected by a correlation between the underlying perceptual processes or states as well as by any parity considerations that enter in when more than one input channel is used. No direct measure of either parity or correlation can be obtained from the single experiment with correlated stimuli. However, independent estimates of performance relevant to the parity criterion of independence are usually obtained by running control experiments with single stimulus variables, just as described above with respect to orthogonal input experiments. These independent measures of performance level then provide a base line against which to compare performance on the joint task.

Usually, these independent measures of performance are used to construct a hypothetical matrix of data based on the assumption of zero correlations; that is, a matrix presumed to reflect process in the organism which cannot be overtly measured, as it is with the complete orthogonal experiment. Then if performance with correlated stimuli is different from that expected from this normative model of zero correlation, the discrepancy in effect is

attributed to the existence of correlation. However, performance on the correlated input task will be affected by either a loss of parity, by perceptual correlation, or both, and this experimental procedure cannot differentiate. A more complete and satisfactory procedure would be to run the additional control for process correlation by obtaining data from the orthogonal stimulus experiment. Given the correlations obtained there, a normative model could be constructed which assumes performance with correlated stimuli to involve the same parity and process correlation considerations as are found in the orthogonal case. If performance is different, then we know that the use of correlated inputs has somehow changed the perceptual task for the subject.

*Correlation: process and/or state.* When process and state correlation with the complete orthogonal experiment was discussed, it was pointed out that state correlation in reduced data matrices can be identified with a process correlation term, and in the five sets of illustrative data matrices used, only one showed state correlation, although four of them showed process correlation. This discussion now continues within the context of the correlated input experiment. What we shall see is that the introduction of correlation in the stimulus variables thoroughly complicates the relation between process and state correlation and makes essentially impossible a simple interpretation of a discrepancy of obtained performance data from a value computed from a normative model assuming perceptual independence.

Assume that there are the same two dichotomous stimulus variables,  $A$  and  $B$ , and that these are now presented in a correlated fashion. Two responses are allowed, one appropriate to each of the two stimuli. The data are then reduced to a correct-incorrect form as before (although the term reduction is now less appropriate than transformation, since it is a  $2 \times 2$  matrix to start). Percentage correct responses should be computed for the ideal case of complete perceptual independence, and for this purpose a decision rule based on data in percentage correct form is assumed, although the decision rule is presumed to exist in the organism and to produce

the manifest data, rather than to exist external to the organism and to be applied to the manifest data. Expected performance for each of the other data matrices is then compared in turn:

The reduced matrix in Table 2, for the perceptual independence case, is the same whether or not correlated stimuli are selected. An assumed internal decision process which uses an inclusive disjunction rule shows that there should be 84% correct when two stimulus variables are used in a correlated fashion. That is to say, when another variable is added, and perceptual independence exists, then percentage correct responses increases from 60% to 84%. There has been some gain in perceptual performance.

The reduced data matrix in Table 4 is also true regardless of the subset of correlated stimuli selected, and this is the matrix in which there is state correlation. Thus performance should be less than the normative value of 84%, and for the inclusive disjunction rule it does indeed drop to 76%. Thus the state correlation has produced lower performance than expected, given independence; we therefore reject the assumption of zero state correlation, correctly.

For the matrix in Table 5, and the other two matrices, new reduced matrices must be formed because they are different for correlated stimuli and furthermore depend on which subset of correlated stimuli we use. Table 12 shows the two possible matrices taken from Table 5. The reduced matrix in 12A is obtained by considering only the first and fourth columns of Table 5 and transforming the data to percentage correct as before, except that after summing the appropriate numbers they are changed to percentages again. Thus the value for  $A+B+$  is the sum for  $A_1B_1a_1b_1$  and  $A_2B_2a_2b_2$  from Table 5, which is 22, but becomes 44 to maintain the numbers in percentage form. The reduced matrix in 12B is obtained from the second and third columns of Table 5, with the cell entries obtained in comparable fashion. Thus the cell entry in Table 12B for  $A+B+$  is the sum of  $A_1B_2a_1b_2$  plus  $A_2B_1a_2b_1$  values in Table 5.

TABLE 12

DATA MATRIX 5 REDUCED TO PERCENTAGE CORRECT FOR THE TWO TYPES OF STIMULUS CORRELATION

Variable	Variable		$\Sigma$
	A +	A -	
A: A and B take same values			
B +	44	16	60
B -	16	24	40
$\Sigma$	60	40	100
B: A and B take opposite values			
B +	28	32	60
B -	32	8	40
$\Sigma$	60	40	100

Note.—“+” is correct and “-” is incorrect. Cell entry is percentage of joint occurrence of correctness for stimulus variables A and B. For an actual experiment, these matrices would be hypothetical, referring to a presumed internal process. Tables 13 and 14 are to be interpreted in similar way. For A:  $U(A_e:B_e) = 0.081$ . Percentage correct for: inclusive disjunction = 76; 50-50 on doubtfuls = 60; conjunction = 44. For B:  $U(A_e:B_e) = 0.084$ . Percentage correct for: inclusive disjunction = 92; 50-50 on doubtfuls = 60; conjunction = 28.

Note that there was no state correlation in the reduced matrix from the complete orthogonal experiment, but that there is substantial correlation (shown as the contingent uncertainty for correctness in the note to Table 12), for each of the two matrices selected for correlated stimuli. Furthermore, the inclusive disjunction rule gives a value for correct responses of 76% in one case and 92% in the other. These two values average out to the value expected for perceptual independence (as shown for the reduced matrix in Table 6), but we would conclude that performance was better than that for independence in one case and poorer in the other. What has happened here is that the selection of stimulus subsets has converted a fairly straightforward joint response bias into two state correlation terms, one positive and one negative, depending on which subset of correlated stimuli has been selected. Thus the particular nature of the joint response bias will produce apparent non-independence in the perceptual state, even though none exists in the complete reduced matrix. The use of correlated stimuli interacts strongly with the nature of perceptual process

TABLE 13

DATA MATRIX 7 REDUCED TO PERCENTAGE CORRECT FOR THE TWO TYPES OF STIMULUS CORRELATION

Variable	Variable		$\Sigma$
	A +	A -	
A: A and B take same values			
B +	36	24	60
B -	24	16	40
$\Sigma$	60	40	100
B: A and B take opposite values			
B +	24	16	40
B -	36	24	60
$\Sigma$	60	40	100

Note.—For A:  $U(A_e:B_e) = 0$ . Percentage correct for: inclusive disjunction = 84; 50-50 on doubtfuls = 60; conjunction = 36. For B:  $U(A_e:B_e) = 0$ . Percentage correct for: inclusive disjunction = 76; 50-50 on doubtfuls = 50; conjunction = 24.

correlation to produce apparent state correlation where none truly exists.

The data from Table 7 are reduced into the two separate matrices in Table 13. In this case, the stimulus selection does not produce apparent state correlation, reflecting the lack of the partial response contingent uncertainty,  $U_{AB}(a:b)$ , as shown in Table 11. However, note that in this case the inclusive disjunction rule gives an 84% value (thus suggesting perceptual independence) for the matrix in which stimulus values are the same, while showing the lower value of 76% for the matrix in which the values differ. This latter result, obtained in an actual experiment, would be interpreted as indicating state correlation. Inspection of the two matrices, however, shows that it is due to the paradoxical result showing 60% errors to B, offsetting the apparent 60% correct with the other selected subset of correlated stimuli. Both of these values are produced only by selection of the subset of stimuli, since in the original matrix there is no correlation between B and response to B.

The data from Table 9 are shown in the two separate matrices in Table 14. This idealized case is, of course, extreme for purposes of illustration, since the data were

constructed on the assumption that  $a$  is perfectly correlated with  $b$ . And yet it better makes the point. Here once again there is a state correlation produced in each of these matrices by the selection of correlated stimuli. It is negative in one case, positive in the other. Furthermore, we again have the paradoxical result of more than chance errors for  $B$  in Table 14B. The inclusive disjunction rule, for these two cases, gives 60% for one matrix (no improvement over a single variable) and 100% in the other, a result which would be difficult to interpret, to say the least.

Thus the selection of a correlated stimulus subset introduces changes in apparent average performance and introduces state correlation as a result of the existence of process correlation not related to state correlation in the complete matrix. The use of the correlated input experimental paradigm truly makes understanding of perceptual independence difficult.

*Decision rule: process and/or state.* We have seen that the particular decision rule presumed to exist in the organism greatly affects the calculated percentage correct responses which is to represent the value for a normative model of independence, and that furthermore the rule used gives differential expectations about performance for the five basic sets of data used as illustrations. That is to say, which nonindependent case will give greater performance depends on the decision rule assumed to be going on in the organism. A still further consideration needs to be discussed and that is whether the decision rule pertains to process or to state.

All of the decision rules discussed so far, and those illustrated in the various tables, are rules pertaining to the organismic state, not the process. Thus, for example, when we speak of an inclusive disjunction rule, this rule is being applied to correctness of responses not to the responses themselves or the perceptual processes which they reflect. The inclusive disjunction rule says that the subject will be correct for the correlated input case if he would have been correct on either of the two stimuli presented alone. This is the type of decision rule used most frequently in models of perceptual independence, and yet

TABLE 14  
DATA MATRIX 9 REDUCED TO PERCENTAGE CORRECT  
FOR THE TWO TYPES OF STIMULUS CORRELATION

Variable	Variable		$\Sigma$
	A +	A -	
A: A and B take same values			
B +	60	0	60
B -	0	40	40
$\Sigma$	60	40	100
B: A and B take opposite values			
B +	0	40	40
B -	60	0	60
$\Sigma$	60	40	100

Note.—For A:  $U(A_c: B_c) = 0.971$ . Percentage correct for: inclusive disjunction = 60; 50-50 on doubtfuls = 60; conjunction = 60. For B:  $U(A_c: B_c) = 0.971$ . Percentage correct for: inclusive disjunction = 100; 50-50 on doubtfuls = 50; conjunction = 0.

it must surely be a questionable procedure, with its implication that somehow the organism makes a joint response based on whether it was right or wrong with respect to the separate input channels. If the organism knows whether it is right or wrong, then it can rectify things. More sophisticated decision rules can be used that are more reasonable (see Eriksen, 1966), but there is a further and more important point to make, namely, that expected outcomes with respect to different assumed underlying processes are rearranged if expected correct responses are calculated with a decision rule based on process rather than on state.

To illustrate reasonably realistically, assume an experiment on detection of presence of two visual stimuli,  $A$  or  $B$ , each of which is present or absent together on any given trial; that is, there are trials on which no stimulus occurs or trials on which two stimuli occur. This is a straightforward redundant stimulus experiment. The subject is required to state whether or not he saw anything on each trial. Then assume that what happens in the organism is reflected variously in Tables 1-10, so that if the subject had been permitted, he would have at times said he saw both stimuli, one and not the other, or neither. But since he is allowed only a dichotomous response, he must

invoke a decision rule. If his decision rule is based on whether he would have been correct, then the consequences of different decision rules for different basic perceptual situations are what has been shown in the various tables.

Suppose, however, that his decision rule is based on what his response would have been, not whether that response would have been correct; that is, his decision rule is process based, rather than state based. The use of the inclusive disjunction rule assumes he will say that he saw a stimulus any time he would have said yes to one and/or the other stimulus. We look at the columns in the complete orthogonal matrix and find what percentage of the time a correct response would have been made. To illustrate, assume the data in Table 1, and that a subscript of 2 means the stimulus has been presented and that the response would have been positive. In Column 1, with neither stimulus presented, the subject would have said he saw a stimulus whenever he would have used at least one "2" response, and would be wrong in each of 16 such cases. Alternatively, he would have been right only 9 times of the 25 times no stimulus was presented, that being the number of times he would have said "no" to both stimuli *A* and *B*. In Column 4, with both stimuli presented, he would have said that he saw a light 21 times of the 25 presentations. The total of correct responses for both no stimulus and the redundant positive stimulus is 30, or 60%.

This value is furthermore true for each of the matrices in Tables 1, 3, 5, 7, and 9. Thus not only is the calculated value considerably lower than one based on a state rule, but it does not differentiate between these various conditions at all. Still further, suppose the decision rule which gives 50-50 on doubtfuls is used, again applied to expected responses, not expected correctness (i.e., half the responses in the second and third rows of the complete matrices are assumed to lead to a positive response). Every matrix again would produce 60% correct responses. Thus neither the rule nor the data matrix gives differential expected correct responses for the correlated stimulus case. And the value expected with correlated stimuli is exactly what can be

expected with just one stimulus variable operating. Perhaps such a result would be surprising if obtained. This result, however, has been obtained in more than one experimental situation (see Garner & Lee, 1962). Thus perhaps it needs more consideration than it has been given.

There are many types of experimental situation in which it makes eminently more sense for the decision rule to be based on process rather than state, and since we have seen how great the discrepancy can be between normative values based on process rules rather than state rules, this distinction requires serious consideration. For example, consider experiments in which speech context provides one source of information and a brief visual or auditory exposure provides the other source of information, and let us consider a realistic illustration. A subject is given the sentence context: "The animal that just walked into the barn is a \_\_\_\_\_." He considers possible alternative responses and decides that "dog," "cat," "cow," "horse," and "pig" are possible responses, and over a large number of such cases he would use all of these five possible responses, but they all exist for him as possible responses in some hierarchy of choice on each occasion. Suppose that the correct word (as decided by the experimenter) is pig and that the subject uses this word 20% of the time. Then the word pig is presented auditorily, but in noise. The subject considers as alternatives "pig," "big," "tick," and "pick." He chooses pig 50% of the time and thus is correct that percentage of the time. Now given these two values for percentage correct for each source of information separately, the inclusive disjunction rule for independence based on perceptual state can be used, with the conclusion that with both sources of information, the subject, or many different subjects, should be correct 60% of the time. If then they are correct more often than this, we conclude that there is nonindependence, with some sort of facilitating interaction between the information inputs.

Consider a reasonable rule based on process, however, which is that the subject will give that response which coincides for the two

possible sets of alternative responses. (In milder form, this is the kind of process described by Tulving and Gold, 1963.) Since only one word coincides, he will use it and will be correct 100% of the time. Thus if we find a result less than this, we will conclude that there is nonindependence, with duplication of information in the two sources.

Once again, perhaps such an expected result seems unreasonable. Yet results close to it have been obtained (Pollack, 1964; Tulving, Mandler, & Bauml, 1964).

*Can independence be tested with correlated stimuli?* When we consider the number of factors about which underlying, not directly testable, assumptions must be made for a normative model of perceptual independence, it is difficult to be sanguine about the value or possibility of asking whether perceptual processes are independent when the experimental paradigm using correlated inputs is used. We can assume parity independence, but not correlation independence, in either process or state form, and we can assume different decision rules in either process or state form. The number of possible combinations is so great that surely a normative model can be found demonstrating perceptual independence, at least of some kind, for any given experimental result. Equally true is that we can disprove the assumption of perceptual independence for any given experimental result. Under such circumstances, the question cannot be very meaningful.

Furthermore, at best, a normative model used with correlated inputs can allow us to reject or accept the concept of independence but would be quite unable to let us know why a particular experimental result is obtained if the independence concept is rejected. As pointed out in the beginning, this property of a normative model is certainly one of its strengths as contrasted to models or theories of the psychological process itself, which are true or false insofar as they correspond to data rather than insofar as they help clarify an experimental result. Thus it is clear that the experimental paradigm using correlated inputs has limited utility in the study of perceptual independence.

## INTERACTION OR INDEPENDENCE

The question of perceptual independence does have some meaning in psychology, particularly when the complete orthogonal experiment is feasible. And perhaps it can have some meaning with redundant or correlated stimulus inputs if there are logical grounds for establishing a more sharply delineated set of concepts pertinent to the problem. Then we would not test for independence as a general concept, but as a much more specific concept.

However, consider briefly the possibility that the basic approach is not meaningful and that other conceptualizations of the perceptual problem might further our understanding more rapidly. In particular, we think that models and experiments aimed at specifying the nature of interactive processes in the organism will probably be more useful than those which are especially concerned with the particular type of interaction called perceptual independence.

### *Where Does Interaction Occur?*

Certainly a major consideration in understanding perceptual interaction is the locus of the interaction. The assumed locus of decision rules makes a large difference in assumptions about perceptual performance with multiple sources of information. Further consideration of possible loci of interaction may help clarify the question of perceptual interaction and may show that some of the apparent contradictions in conceptualization of independence in terms of process separability are not necessarily as confusing as at first glance.

To illustrate, assume two extreme cases of locus of interaction, and consider how these assumed loci of interaction will affect the kind of apparent perceptual independence. First, consider two channels which are literally independent in the sense that they are completely separate organisms. One organism is given stimulus variable *A* and it responds with response variable *a*; the other organism is given stimulus variable *B* and it responds with response variable *b*. Variables *A* and *B* are then presented in correlated fashion, and the outcomes from *a* and *b* are independent. There can actually be four response outcomes



from the two organisms and the distribution of outcomes for stimulus  $A_1B_1$  and for stimulus  $A_2B_2$  can be seen. We will assume the distributions from Table 1 to hold, and from these we (the experimenters, that is) make a dichotomized decision. Given on our part knowledge of the outcomes, without knowledge of the stimuli, there is no decision rule we can use which will give us greater than the 60% accuracy shown to hold with an assumed process rule for the organism itself.

Is such an outcome from a single organism ever realistically to be expected? If in fact the two channels of information are so completely separated that a subject can apply a decision rule only to his responses, the answer is certainly yes. And there are experimental results which show little or no gain due to the use of redundant stimulus information. For example, for simple multiple visual stimulation, Garner and Lee (1962) found no gain in discrimination accuracy with increased number of elements; Eriksen and Lappin (1965) found small gain as long as the subject was uncertain of position for the smaller number of stimuli. Both of these results have been corroborated by Garner and Flowers (1969). So there are experimental results which suggest that the interaction occurs very late in the total system between input and output.

But of interest here is the relation of such results and the interaction concept to that of independence. Usually we assume a gain in performance with independence, because the two or more sources of information, being processed independently, add to improve discrimination. If the processes are correlated, thus duplicating information at some stage, little or no gain is expected. Yet this conceptualization of interaction only at the response level carries with it the idea of complete independence of the information channels. Thus the use of the independence idea leads to incorrect assumptions about expected performance, whereas the use of the interaction idea at a specified locus does not.

Now consider the opposite end of the continuum. Double stimuli, correlated, but in the same organism, are presented on the same retina, on the same location of the retina, and at the same time. If ever a condition where

nonindependence exists was produced, this is it. Yet what will such a result show? That there is a gain in discriminability, because the two stimuli will add in energy. This is what Garner (1965) called *energetic redundancy*, because there is an actual or potential summation of stimulus energy.

To refer once again to the Garner and Lee (1962) experiments, they found no gain in discrimination when stimulus elements were added at the same time but in different retinal locations. But they found considerable gain when the duration of the stimulus was increased on the same retinal location. Within the present context of discussion, this differential result would be interpreted as being due to the locus of interaction, large gain occurring when the locus is at the sensory periphery, and no gain occurring when the locus is at the response periphery.

Again of interest here is the relation of the concept of interaction to that of independence. Since literal spatial and temporal duplication of stimuli should produce nonindependence, then we should expect little gain in performance with duplicated stimulus elements. But the opposite effect occurs. Thus it no longer seems clear that independence is a very useful concept, whereas locus of interaction is.

Notice that the assumed locus of interaction will have great effect on the kind of interaction which can occur. In our example of a decision process occurring after the response, then the interaction must occur with the discrete responses; on the other hand, at the sensory end, the interaction can occur with a continuum. Thus models with continuous metrics are appropriate. Models using  $d'$  are of this form (see Taylor, Lindsay, & Forbes, 1967; Ulehla, Halpern, & Cerf, 1968), and the underlying process model assumed by Garner and Lee (1962) was a variance model also.

Morton (1967) has shown, using the  $d'$  metric, that the locus of interaction affects the normative outcome to be expected with systems otherwise preserving independence. He showed that the expected gain in discrimination with correlated inputs is greater when the information was combined prior to a decision

mechanism than it was when a combinatorial rule is applied after decisions are made. Thus the importance of the locus of interaction in no way depends on the use of the simple probabilistic combinatorial rules discussed.

As a further illustration, Morton (1969) has elaborated a Logogen Model for word recognition which deals with multiple sources of information. In a "logogen," evidence relevant to a particular response is collected indifferently with respect to the source of information. The correctness of the response actually made is a function of the amount of evidence for that response and the amount of evidence for other responses. For the case of a verbal context followed by the noisy presentation of a stimulus word, the model predicts interaction between the two sources as

$$\text{logit}P_{sc} = \text{logit}P_s + \text{logit}P_c + \text{constant} \quad [10]$$

where  $\text{logit } P = \log [P/(1 - P)]$ , and  $P_s$ ,  $P_c$ , and  $P_{sc}$  are, respectively, the probabilities of being correct with stimulus alone, with context alone, and with both sources of information. This relation, which gives a reasonable account of the data, is clearly in the form of a performance-parity statement. The important point of the above equation, however, is that the logit transformation required to make the parity statement is the result of a particular assumed locus of interaction and an assumed nature of the decision rules. Other loci of interaction and other decision rules would require different transformations of the data in their parity statements. Thus models which explicitly take into account differences in locus of interaction and in decision rules applied can be used and will hopefully allow greater clarification of the underlying processes involved.

This idea of locus of interaction rather than independence is useful also in understanding some of the results found in multidimensional distance judgments, where the interstimulus difference seems to depend on the nature of the stimulus dimensions and their relations to each other (e.g., Handel, 1967; Hyman & Well, 1968; Shepard, 1964). And concepts such as Lockhead's (1966) integrality and

Shepard's (1964) separability perhaps are more meaningfully phrased in terms of locus of interaction of the stimulus dimensions, with locus of interaction being more toward the receptor periphery with integral dimensions.

## CONCLUSIONS

While the aim in this paper has been less to draw definitive conclusions about the nature of perceptual independence and how to investigate it than to engage in a discussion of the rationale of the problem, certainly some fairly definite implications do emerge:

1. The concept of perceptual independence is not a unitary concept and cannot be studied as such. There are too many variations in basic definition, type of normative model, and experimental paradigm for it to be so.

2. The only experimental paradigm which is reasonably effective in dealing with perceptual independence is the complete orthogonal experiment (plus some controls for parity), since several different terms involved in the independence concept can be separately identified.

3. Even with restricted definitions of perceptual independence, experimental paradigms using correlated inputs are not effective in dealing with the problem. So many assumptions must be made about type of independence and decision rules for obtaining composite responses that some normative model of independence can be found to satisfy almost any experimental result.

4. While the concept of state independence is potentially valuable, it is clearly interrelated with process independence and if possible should not be used without consideration of process correlation and parity. Certainly reduction of data to test for state independence destroys much potential information about perceptual independence and can obscure results clearly indicating nonindependence.

5. Our understanding of perceptual processes will be advanced more effectively with models designed to elucidate the nature of an interaction process, particularly the locus of interaction, than with models which deal only with that kind of interaction called independence.

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